EXOPLANETS AND ω-PRECESSION

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Abstract. Radial velocity measurements of the exoplanet systems HD41004b, WASP-14b and XO-3b need to be accurate to several m/s in order to pick up relativistic precession over a year.

1. Introduction

The velocity amplitude $K \sim \max(v_{\mathrm{l.o.s}})$ of the star is

$$K \sim \frac{M_p \sin I}{\sqrt{a_g}} \sim \frac{M_p}{\sqrt{M_p + M_s}} \frac{\sin I}{\sqrt{a}}. \tag{1}$$

Now since

$$\frac{P^2}{a^3} = \frac{4\pi}{M_p + M_s} \frac{1}{G}, \tag{2}$$

$$K P^{\frac{1}{3}} \sim \frac{M_p}{(M_p + M_s)^{\frac{2}{3}}} \tag{3}$$

2. Sources of precession

General relativity predicts that the argument of pericenter ω of a freely-falling bound 2-body system advances by

$$\Delta\omega_{GR} = \frac{6M\pi G}{c^2 a (1 - e^2)} \text{rads/revolution} \sim 0.2 \left(\frac{M}{M_\odot}\right)^{2/3} \left(\frac{P}{\text{day}}\right)^{-5/3} \text{deg/yr}. \tag{4}$$

Because orbital precession is in-plane, of all the Keplerian elements, only ω is affected. Our aim here is to investigate to what accuracy ω can be determined. The shape of the radial velocity curve of an star harboring an exoplanet changes over time due to the change in ω. In this text, we ask how well can we recover ω, and therefore to what extent one can detect a $\Delta\omega$ shape change. See Figure 1. for an example of how the radial velocity curve changes shape over significant time periods due to GR-induced ω-precession. Precession induced by another planet is

$$\dot{\omega}_{\text{pert}} \approx \left(\frac{P}{\text{day}}\right)^{-1} \left(\frac{a}{a_2}\right)^{3} \left(\frac{M_*}{M_\odot}\right)^{-1} \left(\frac{M_2}{M_\oplus}\right) [\text{deg century}^{-1}]. \tag{5}$$

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Now the ratio of (5) and (4) and is

$$\frac{\dot{\omega}_{\text{pert}}}{\omega_{GR}} = 3\left(\frac{a}{b}\right)^3 \left(\frac{M_*}{M_\odot}\right)^{-2} \left(\frac{M_b}{M_\oplus}\right) \left(\frac{a}{0.05\text{AU}}\right), \tag{6}$$

where b is the semi-major axis of the perturbing planet, and its mass M_b. Now the velocity amplitude of the star due to the perturbing planet is

$$K_b = 30 \left(\frac{M_*}{M_\odot}\right)^{\frac{1}{2}} \left(\frac{b}{1\text{AU}}\right)^{-\frac{1}{2}} \left(\frac{M_b}{M_*}\right) \text{km s}^{-1} \tag{7}$$

The oblateness of a gravitating body gives rise to a Newtonian potential term $\sim r^{-3}$. This induces a prograde precession,

$$\Delta\omega_{oblate} \sim s\frac{\Delta\omega_{GR}}{a\left(1-e^2\right)}, \tag{8}$$

where s is the oblateness of the star. Under the assumption that $s = s_\odot$, the second panel of Figure 2. plots (4) against (8) for all known exoplanet systems with $\Delta\omega_{GR}$ greater than Mercury.

If the accuracy the radial velocity measurements is such that the accuracy in the recovered value for ω is greater than the expected precession over a certain time interval, then a suitable observing strategy could pick up the precession: the value for ω, recovered from two different observing runs, at say, a year or more apart.

Measuring relativistic precession on exoplanet systems is itself a rewarding goal. Beyond this however, relativistic precession may be used as a tool to break the $M \sin I$ degeneracy, and allow the determination of the inclination of non-transiting exoplanets.

3. Examples

We have selected four exoplanet systems as case studies [1, 2, 3, 4]. We have chosen them for their short period (and therefore high rate of precession), and their non-zero eccentricity[1]. See the left panel in Figure 2 in order to compare our choices to the other less-desirable known exoplanet systems.

(1) WASP-12b is a transiting extrasolar planet with an extremely short period of 1.1 days. Of our selected examples, WASP-12b, is expected to experience the largest relativistic precession, at 0.2 degrees per year. However, as will be argued, in wishing to pick up relativistic precession, the low RV amplitude of 0.02kms^{-1} demands accuracy beyond what currently available spectrometers are able to deliver.
(2) HD41004 B, possibly a brown dwarf with a 1.3 day period, orbiting a visual double system is certainly the most complicated system of our examples. The RV amplitude of 6.1kms^{-1} is the largest of our set.
(3) WASP-14b is a short-period transitor with a 2.2 day period.
(4) Of our selection, XO-3b has the highest eccentricity, yet longest period, at 3.2 days.

[1]Note that if the eccentricity of a system is zero, then the argument of pericenter is not defined. In this case, while the orbit still precesses, its shape remains unchanged, and so detection is more challenging.

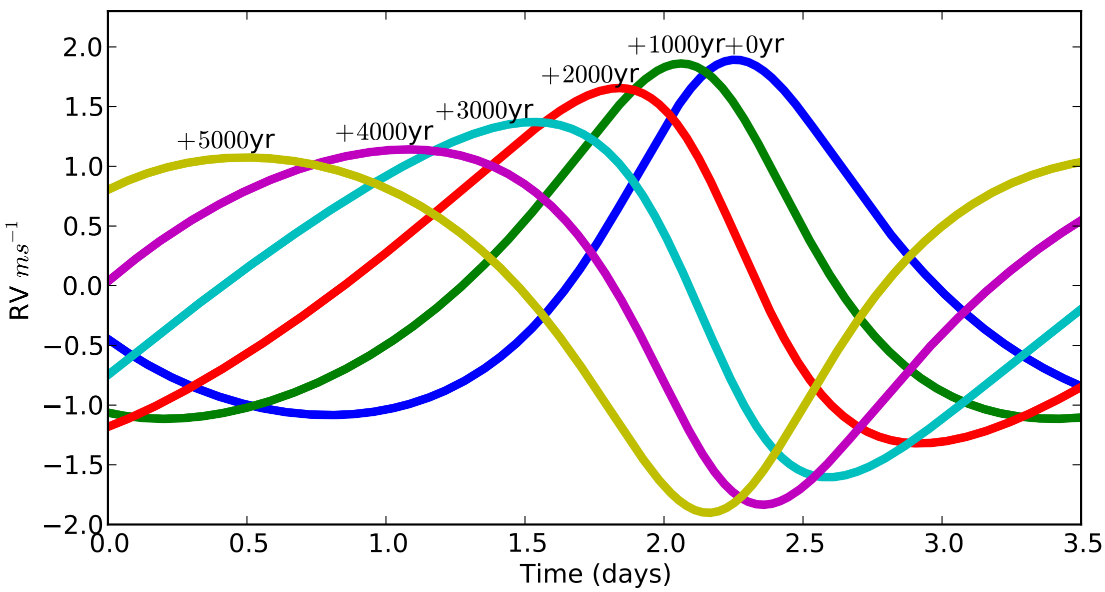

FIGURE 1. This diagram shows how the shape of the radial velocity curve of XO-3b [4] changes over time due to relativistic precession. Every 1000 years, the argument of pericenter advances by 40 degrees.

	K kms^{-1}	e -	M M_{sol}	P days	ω deg	I deg	$\Delta\omega_{GR}$ deg/year	ref
1	0.02	0.2	1.28	1.09	-74	83	0.2	[1]
2	6.1	0.08	0.4	1.3	178	-	0.07	[2]
3	1.0	0.09	1.3	2.24	-106	84	0.06	[3]
4	1.5	0.28	1.4	3.2	346.3	90	0.04	[4]

For each of these examples, we fit to the radial velocity curve using the Keplerian model in the barycenter. The fitted parameters are amplitude K, eccentricity e, period P, inclination I, heliocentric velocity i, and the argument of pericenter ω. For each of the four systems, we generate mock data consisting of 50 measurements at fixed accuracy taken over a 10-period interval. For each realization, a quasi-Newtonian optimization algorithm works to minimize the χ^2. We do this for many realizations, which gives the 1-σ spread of the parameter values after fitting. Figure 3 shows the mock data for each system, and Figure 4. the accuracy in the recovered ω as a function of dispersion in the data.

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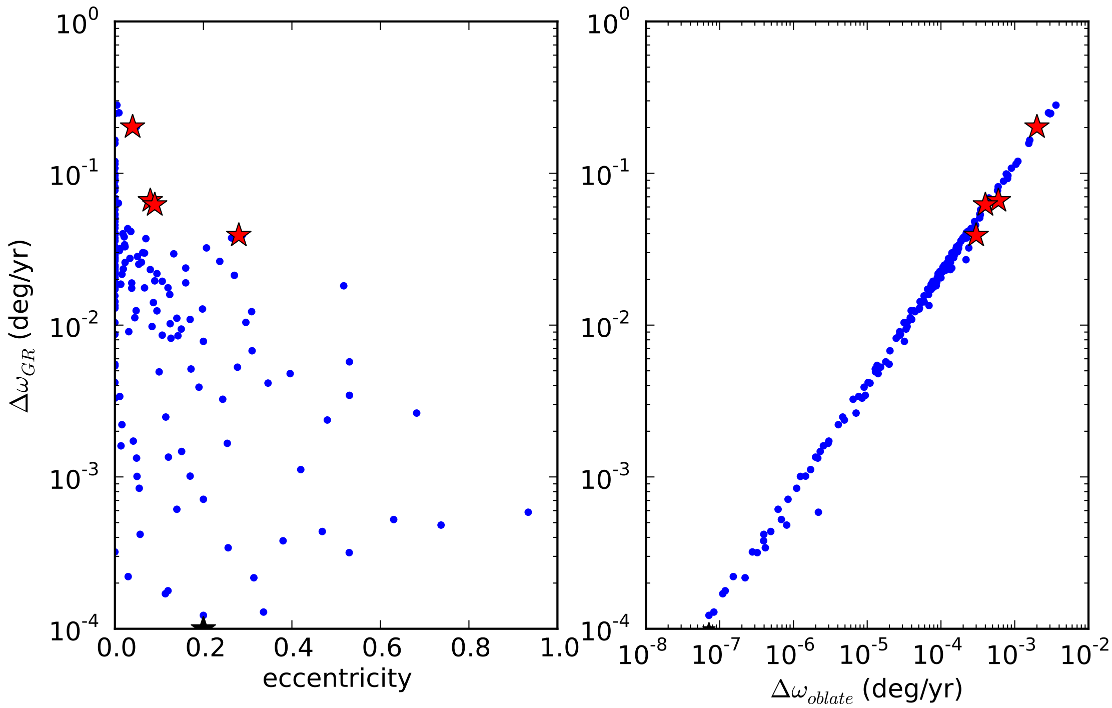

FIGURE 2. Each mark denotes a known exoplanet, save for the black marker peeking out from the bottom denoting Mercury. The red star markers mark the exoplanet systems we have selected as examples. The vertical axes on these plots show the expected relativistic precession rate. The left panel shows this against the eccentricity distribution. The right panel shows the GR-precession rate against an estimate for the precession rate due to the oblateness of the star, were the star to have the same degree of oblateness as the Sun. Naturally, were the angular momentum and therefore oblateness of a star large, and oblate-induced precession begins to dominate, disentangling $\Delta\omega_{GR}$ from $\Delta\omega_{oblate}$ could be tricky.

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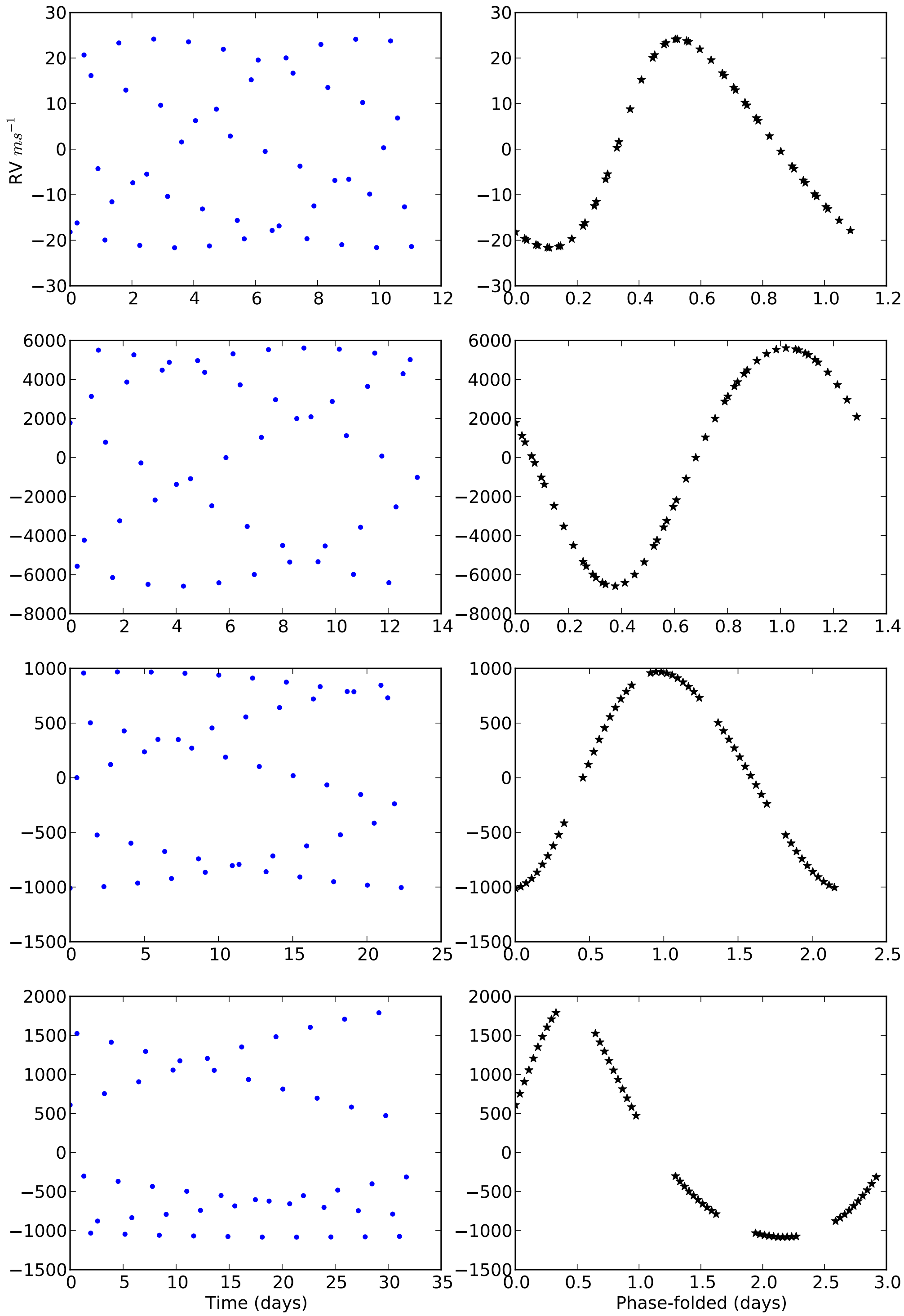

FIGURE 3. Radial velocity data for each of our four cases. It is to such mock data to which we fit in order to determine the uncertainties in the recovered parameter values.

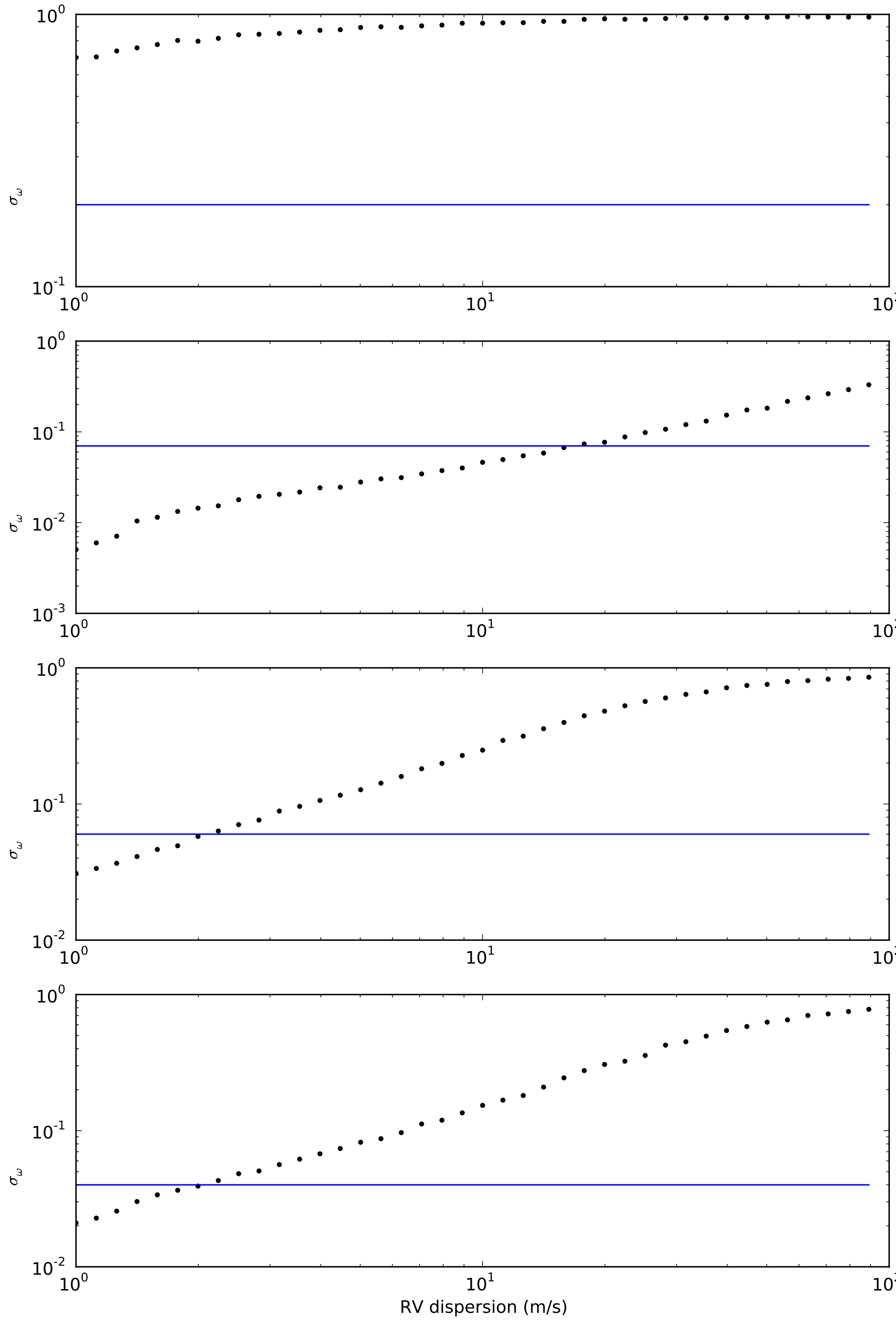

FIGURE 4. The standard deviations in ω as a function of the dispersion in the 50 data points for each of our 4 cases. The horizontal blue line indicates the amount that ω is expected to precess due to relativity in one year. For three of the four cases, two observing runs, performed one year apart, each with 50 data points at accuracy below the blue lines, should suffice in observing relativistic precession.

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